

1 **Methods for shear strengthening of thick concrete slabs**

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3 **Abstract:**

4 This paper presents different strengthening techniques to improve the shear capacity of existing
5 thick concrete slab structures that were constructed without shear reinforcement. Reinforcing
6 bars are installed into vertical drilled holes and anchored with epoxy adhesive to increase the
7 shear capacity. Experiments on retrofitted beams, representing slab strips, showed that all of the
8 strengthening techniques investigated resulted in increased shear capacities. The shear failure
9 mechanisms of the strengthened beams showed that, as expected, current evaluation methods for
10 elements with conventional, well-anchored stirrups can lead to an overestimation of the shear
11 capacities. The efficiency of the strengthening techniques is strongly influenced by the
12 performance of the end anchorage of the drilled-in bars. The effectiveness of the epoxy-bonded
13 bars is a function of their embedded length and they can, in some cases, debond before they
14 reach their yield strength. By using the maximum bar spacing required by the Canadian Highway
15 Bridge Design Code or the AASHTO-LRFD design specifications, bonded shear reinforcing bars
16 may debond and offer poor performance. A maximum transverse reinforcement spacing criterion
17 is therefore suggested for the added bonded bars.

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19 **Introduction**

20 Concrete thick slab bridges typically have spans ranging from 6 m to 25 m (20' to 80') with a
21 structural slab thickness of 300 mm to 1500 mm (12 in. to 60"). For this simple structural
22 system, the thick slab is designed to carry all loads and therefore no support beams (girders) are
23 required. For the design of thick concrete slabs, it is often assumed that the shear capacity
24 provided by the concrete is sufficient to resist the shear and therefore, no shear reinforcement is
25 required. On September 30th, 2006, the Concorde overpass (Laval, Quebec, Canada) collapsed,
26 killing five people and injuring six others (Fig. 1). Even though the original design complied
27 with standards at the time of construction, a shear failure in the cantilever region of the
28 supporting concrete thick slab led to collapse (Johnson et al. 2007; Mitchell et al. 2011). This
29 shear failure occurred in the 1200 mm (48 in.) thick slab that did not contain any shear
30 reinforcement. The shear failure was very brittle and lead to the sudden collapse of one-half of
31 the overpass structure. For the Concorde overpass collapse, the investigation has shown that
32 concrete degradation with time resulted in the propagation of inclined cracks, followed by a
33 brittle shear failure. That tragic event raised questions concerning the safety of many aging thick
34 concrete slab bridges without shear reinforcement. Moreover, the investigation showed that the
35 provision of the minimum amount of shear reinforcement recommended by the 2014 Canadian
36 Highway Bridge Design Code S6 (CHBDC) (CSA 2014a) would have prevented the Concorde
37 overpass collapse. Because of the deficiencies in shear of this type of construction, practical
38 methods incorporating shear reinforcing bars into thick concrete slabs have gained wide interest.
39 Some shear strengthening methods have already been studied on narrow beams. The addition of
40 near surface mounted rods (De Lorenzis and Nanni 2001; Dias and Barros 2008) and the addition
41 of external carbon fiber reinforced plastic (FRP) laminates (Adhikary and Mutsuyoshi 2006;

42 Teng et al. 2009) have been proven successful to increase shear capacity. However, the
43 anchorage of such shear reinforcement on either side of a concrete beam section raised question
44 of their efficiency on the full width of wide structural elements like thick slab bridges. On one
45 hand, thin slabs strengthened in shear with bonded inclined drilled-in rods have shown their
46 efficiency to increase punching shear capacity (Fernández Ruiz et al. 2010). On the other hand,
47 very few studies were performed on the strengthening of existing thick slabs where the well
48 known “size effect” (Godat et al. 2010; Collins et al. 2008) will influence the shear capacity.
49 This paper presents shear strengthening techniques that can be used on existing concrete thick
50 slab structures to improve the shear capacity (Fig. 2). The loading tests performed as well as the
51 comparison between shear capacities of concrete thick slab strips (beams) strengthened and
52 unstrengthened in shear are presented. The techniques investigated consist of placing reinforcing
53 bars into pre-drilled vertical holes with epoxy bonding. The performance of this system has been
54 examined through experimental tests. The responses of the strengthened beams are compared to
55 tested reference beams, with conventional stirrups and without stirrups.

56 **Description of Post-Installed Shear Strengthening methods**

57 Fig. 2 shows the installation of two strengthening techniques. The first method (Fig. 2a) consists
58 of filling drilled holes with high-performance epoxy adhesive to bond the full length of steel bars
59 to the concrete. For this case, the holes are drilled from the top surface and the bars are inserted
60 into the epoxy-filled holes from the top. The alternative method (Fig. 2b) consists of introducing
61 the epoxy-bonded shear reinforcement from both the top and bottom slab faces, in order to
62 provide longer bar embedded lengths near the bottom surface.

63 **Experimental Program**

64 Two series of three point loading tests were performed for a total of 15 beams representing thick

65 slab strips (beams) which were designed to experience shear failures. All simply supported
66 beams (4000 mm span) have a rectangular cross section of 610 mm width, b_w , and constant
67 heights, h , of 450 mm or 750 mm. Details of the beams are presented in Table 1 and Fig. 3.
68 Three reference specimens, (U1, U2, and U3), without shear reinforcement were also tested.
69 These beams are not shown in Fig. 3 but they have the same overall dimensions and flexural
70 reinforcement as the shear strengthened beams B1, B2 and B3.

71 *Description of the 1st test series*

72 Beam specimens U1, U2 and U3, and their companion beams with post-installed epoxy bonded
73 bars B1, B2 and B3 were designed to study the effectiveness of the bonded shear reinforcement
74 on the beam shear capacities. The spacing ratio, equal to the spacing of the bars, s_v , divided by
75 the effective shear depth, d_v , of the post-installed shear reinforcement is close to the maximum
76 allowed by North American Standards ($k_{v,max} = s_{v,max} / d_v = 0.75$ and 0.80 for CHBDC S6 (CSA
77 2014a) and AASHTO (2012) respectively). Beams B1 and B3 were strengthened using 15M
78 reinforcing bars whereas beam B2-1 and B2-2 were strengthened with 10M reinforcing bars
79 (refer to Table 2 for the bar area, A_b , and the bar diameter, d_b). These post-installed bars were
80 introduced into 14.3 and 19.1 mm diameter holes for the 10M and 15M bars, respectively, and
81 bonded to the concrete with epoxy adhesive (Fig. 2a). The longitudinal reinforcement ratio ρ is
82 presented in Table 1 and, for this first series of beams, 25M bars were used for longitudinal
83 tension reinforcement.

84 *Description of the 2nd test series*

85 Specimens of the second series, which were all strengthened in shear, provide results to compare
86 different reinforcing methods. Beams B4, B5 and S1 have transverse reinforcement spaced such
87 that the ratio $k_v = s_v / d_v$ was 0.61. Beam S1 is a typical reinforced concrete beam with 15M

88 conventional stirrups. Beams B4 and B5 were strengthened with vertical 15M epoxy-bonded
89 bars post-installed into 19.1 mm diameter holes. The post-installed bars in beam B4 were
90 installed from the top face of the beam, whereas they were installed from both top and bottom
91 faces for beam B5 (Fig. 3b). For this series, 30M bars were used for longitudinal tension
92 reinforcement in the beams.

93 ***Material Properties***

94 The average compressive strengths of the concrete, f'_c , presented in Table 1 were determined at
95 an age at which the beams were tested. The maximum aggregate size of the concrete, a_g , was 19
96 mm for all specimens and the concrete density, γ_c , is presented in Table 1. The steel reinforcing
97 bars yielding strength, f_y , and ultimate strength, f_u , are given in Table 2. For all bars, the
98 Young modulus, E_s , was taken as 200 GPa. A commercially available epoxy adhesive was used
99 for the bonded anchorage. The bond strength, τ_b , of the epoxy adhesive was estimated as
100 (Fernández Ruiz et al 2010, ETA 2013):

$$101 \quad \tau_b = 18.7 \left(\frac{f'_c}{20} \right)^{0.1} \quad \text{in MPa units} \quad (1)$$

102 The development length, ℓ_d , of the bonded bars with epoxy adhesive can therefore be evaluated
103 from Eq. (2). It varies between 90 and 97 mm for the 15M bars and is about 62 mm for the 10M
104 bars used for beam B2.

$$105 \quad \ell_d = \frac{d_b f_y}{4 \tau_b} \quad (2)$$

106 ***Testing procedure and measurements***

107 All beams were tested under three point loading. Fig. 4 shows the experimental setup for a 750
108 mm deep beam loaded at mid-span. For the 450 mm deep beams, the loading is applied at one-

109 third of the span only (three point loading). The loading was applied at a rate of 10 mm/h and
110 beam deflections were measured at the loading location. Strain gages (red points in Fig. 3) were
111 used to measure strains in the shear and longitudinal reinforcing bars. LVDTs (Linear Variable
112 Differential Transformers) were installed on the side faces of the beams at mid-depth to measure
113 shear crack width development. A crack comparator was used to measure crack widths. After the
114 tests, concrete core samples were extracted from the beams and some beam sections were cut to
115 examine the anchorage quality of the bonded bars.

116 For the second test series, after the occurrence of shear failure, the beams were strengthened with
117 steel clamping devices as shown on the right hand side of Fig. 4. This allows the reloading of a
118 beam (40 mm/h) until the failure of the other half of the beam (left hand side of Fig. 4). To
119 distinguish between loading and reloading tests of the same beam, these reloaded beams are
120 identified as beams “R”, such as: B4-1R, B5-1R and S1-1R.

121 **Test Results**

122 Table 1 provides a summary of the test results. In this table, the values of the experimental shear
123 force, V_{exp} , (including effects of beam self-weight) and the beam deflection, δ_{exp} , measured at
124 the loading location (see Fig. 4) are given at failure. The applied shear versus δ_{exp} and the critical
125 crack width, w , are shown in Fig. 5. Due to failure of the data recording system, no data is
126 available for beam B1-2. Fig. 6 shows the cracking patterns for the half-portion of the beam
127 where the failure occurred. The critical failure cracks are shown with a bold line while the lighter
128 lines show other cracks having smaller crack widths.

129 **Behavior of slab strips**

130 *Unstrengthened beams (Beams U1, U2 and U3)*

131 As expected, beam specimens U1, U2 and U3 had only minor diagonal cracking up to the

132 maximum failure load. Shear failure occurred after the sudden formation of a critical inclined
133 crack and a horizontal splitting crack along the longitudinal reinforcement. These failures
134 occurred suddenly, with little or no warning. The shear strengths of beams U1, U2 and U3 were
135 324, 291 and 342 kN, respectively. The post-failure resistances were about 100 and 160 kN for
136 beam specimens U1 and U2, respectively. Beams U3 showed almost no post-failure resistance.
137 These tests demonstrate the danger associated with the sudden shear failure mode of concrete
138 thick slab structures without shear reinforcement.

139 ***Beam with conventional stirrups (Beam S1)***

140 The main critical shear crack leading to failure was visible on both sides of beam S1 at a shear of
141 about 525 kN and this critical crack progressed slowly during the remaining loading. Beam S1-1
142 failed at a central deflection of 10.7 mm and a corresponding shear of 726 kN. With a post-
143 failure resistance of 500 kN (decrease of 35%), beam S1-1 exhibited a larger residual shear
144 capacity than the other beams. The reloaded beam test S1-1R was stopped before the shear
145 failure because of the yielding of the steel clamped assemblies used on the other half span. The
146 shear capacity of the beam test S1-1R is therefore higher than 809 kN (see Table 1).

147 ***Beams with bonded shear reinforcing bars (Beams B1, B2, B3, B4)***

148 The beams strengthened with drilled-in, bonded shear reinforcing bars exhibited rapid
149 propagation of a wide diagonal crack with a significant decrease of the beam stiffness. For
150 example, for beam B4-1, it can be observed in Fig. 5 that increasing the shear force by 34 kN
151 (508 to 542 kN) resulted in an increase in the central deflection by 1.9 mm (4.6 to 6.5 mm). For
152 this beam, a large crack of width 1.5 mm propagated from the support to the loading location,
153 crossing reinforcing bars R3 and R4 (see Fig. 6 for bar locations). For comparison, the diagonal
154 crack width in beam S1-1 at 525 kN was 0.3 mm and no cracking was visible crossing

155 reinforcing bars at location R3. For beams B1 and B3, a large critical diagonal crack appeared at
156 425 and 412 kN, respectively. Thereafter, new diagonal cracks appeared during the loading and
157 the maximum shear capacity of beams B1, B3 and B4 were 471, 498 and 756 kN, respectively.
158 By comparison with the unstrengthened beam specimens U1 and U3, the shear capacity of
159 strengthened beams B1 and B3 increased by about 45% and their deflections at failure increased
160 by 79% and 146%, respectively.

161 For beam B2-1, no shear crack was visible before the maximum shear capacity of 288 kN was
162 reached at a 6.8 mm deflection. At this load level, a diagonal crack appeared suddenly and the
163 shear force was maintained below the maximum capacity, at a shear of 285 kN, until a sudden
164 loss of the beam capacity, at a deflection of 8.4 mm. For beam B2-2, the shear force was
165 maintained after the propagation of the critical diagonal crack at a shear of 289 kN. The failure
166 of beam B2-2 occurred at a displacement of 9.8 mm and a shear of 315 kN. Compared with the
167 other beams with drilled-in bonded bars, beam B2 showed no additional shear cracking after the
168 propagation of the large diagonal crack. While the strengthening of this beam resulted in an
169 increase in shear capacity of only 4%, the maximum deflection is 25% higher than those
170 measured in the unstrengthened beams U2-1 and U2-2.

171 As expected, the failure modes of beams B1, B2 and B3, with bonded reinforcing bars, are less
172 brittle: beams B1, B2 and B3 showed signs of their pending failure with large diagonal cracks
173 and higher deformation and post failure capacities than their unstrengthened companion
174 specimens U1, U2 and U3. Likewise, beam B4 showed a similar post failure capacity of 450 kN
175 and a deflection capacity of 12.0 mm as beam S1 with conventional stirrups (500 kN and 10.7
176 mm).

177 ***Beam with overlapped bonded shear reinforcing bars (Beam B5)***

178 Beam B5 with overlapped bonded bars shows more diagonal cracks than the other beams with
179 drilled-in bonded bars. Initially, the progression of most of the diagonal cracks was controlled in
180 the overlapped portion of the shear reinforcing bars. As the load increased, new diagonal cracks
181 appeared in this region. Before the failure, the development of a wide diagonal crack suggested
182 an imminent shear failure. The failure of beam B5-1 is very brittle and the concrete crushed in
183 the compression zone at a deflection of 15.6 mm and a shear of 942 kN. The reloaded beam B5-
184 1R (tested after clamping the failed end) may have been weakened by the very brittle failure of
185 the beam B5-1 and the crushing of the compression zone resulting in a lower shear capacity of
186 823 kN. Therefore, the experimental capacity obtained for the reloaded beam B5-1R was not
187 used for comparison purposes.

188 **Observation of internal cracking**

189 After testing, the beams were dissected to enable inspection of the internal shear cracking and to
190 see the intersection of this cracking with the added shear reinforcing bars. Fig. 7 shows a view of
191 the inside of beam B5-1R that was strengthened with overlapping bars drilled-in from the top
192 and bottom and then bonded with epoxy. The gaps between the bottom end of bar R2t and the
193 top end of bars R3t and R4t indicate that slippage of the bars had occurred due to debonding.
194 This slippage was most apparent at locations where the shear crack resulted in short embedment
195 lengths of the drilled-in bars.

196 **Behavior of shear reinforcing bars**

197 Fig. 8 shows the average of the measured strain in two stirrup legs, ε_{sv} , versus the central
198 deflection of the beams. The yield strain ($\varepsilon_y = f_y / E_s$) of the shear reinforcing bars is shown as a
199 dashed horizontal line. Table 3 shows the embedded lengths and the distance between the strain
200 gage and the main diagonal crack (L_{tc}). For beam S1-1, the values of L_{tc} for stirrups R2, R3 and

201 R4 are 128, 30 and 133 mm, respectively. The bar embedded length ℓ_e for the drilled-in bars is
 202 taken as the shortest length between the main diagonal crack and the bar extremity. The
 203 calculated bar development length, determined from Eq. (2), is shown. The maximum stress in
 204 the bar layer $f_{sv,calc}$ is determined according to the bond strength determined from Eq. (1) and is
 205 limited by the bar yielding strength. A linear stress-strain relationship, given by Eq. (4) is used to
 206 determine the experimental bar stress $f_{sv,exp}$. The maximum values are given in Table 3 according
 207 to the maximum bar strain shown in Fig. 8.

$$208 \quad f_{sv,calc} = \frac{4\tau_b \ell_e}{d_b} \leq f_y \quad (3)$$

$$209 \quad f_{sv,exp} = E_s \varepsilon_{sv} \leq f_y \quad (4)$$

210 As seen in Fig. 8, strains in the bonded shear reinforcing bars only occurred after the shear
 211 cracking load was reached. For beam B5-1 (Fig. 8), the propagation of two diagonal cracks
 212 resulted in increased bar strains at a deflection of about 2.5 mm. The first diagonal crack
 213 intercepted the bars at location R2b (see Fig. 6) and its propagation stopped, while the second
 214 crack crossed the bars at locations R3b, R3t and R4t. Some beams with shear reinforcement
 215 failed shortly after the yielding of one set of added reinforcing bars. The force that can be
 216 developed in each bar is a function of the bar embedment length defined by the location of the
 217 diagonal crack. When the diagonal crack intercepts a reinforcing bar close to one of its ends, the
 218 resulting embedded length ℓ_e could be shorter than ℓ_d and therefore debonding would occur
 219 without reaching f_y . For example, it can be seen in Table 3 and Fig. 8 that the diagonal crack
 220 occurred near the very end of the bar at location R2 of beam B1-1 resulting in a short embedded
 221 length and consequently, this bar debonded.

222 In interpreting the strain readings, it is important to consider the fact that the reinforcing bar

223 strain will be at its highest at a crack location. Thus, even if $\ell_e \geq \ell_d$, it can be seen in Fig. 8 and
224 Table 3 that some of the measured strains in the shear reinforcing bars were below ε_y . However,
225 it can be expected that these bars had reached their yield strain at crack locations.
226 The debonding of some bonded bars can also be seen in Fig. 8. For example, the bars at location
227 R2 in beam B1-1 experienced increasing strains until a maximum of 1659 microstrain ($f_{sv,exp} =$
228 332 MPa) at $\delta_{exp} = 9.9$ mm. This was followed by decreasing strains until 652 microstrain ($f_{sv,exp} =$
229 130 MPa) at the beam failure ($\delta_{exp} = 13.7$ mm). It can be seen in Table 3 that the maximum
230 predicted bar stress $f_{sv,calc}$ of 242 MPa from Eq. (3) underestimates the experimental value of 332
231 MPa.

232 For beam S1-1 with conventional stirrups, the bars at locations R2 and R3 reached their yield
233 strength at a central deflection of 5.7 mm. This is followed by a large increase in the strain of the
234 bars at location R4, reaching a maximum strain 2040 microstrain (408 MPa) at a deflection of
235 9.9 mm. While the strain gages on the bar at location R4 showed strains below the yield strain, it
236 is possible that this reinforcement yielded at the crack location. For this case, $L_{tc} = 133$ mm and
237 hence these bars may have reached f_y . Compared with beam B4, the conventional stirrup legs in
238 beam S1 are well anchored at both extremities. They cannot slip like the drilled-in bonded bars
239 and therefore, they offer better control of the diagonal cracking. Thus, if the diagonal crack
240 crosses a stirrup near the extremities of the stirrup legs, they are still capable of developing their
241 yield capacity.

242 **Comparison with strength predictions and discussion**

243 The predicted shear capacity V_{calc} was determined at a distance d_v from the edge of the loading
244 plate. The shear design provisions of the Canadian Standards Association (CSA) standard A23.3

245 (CSA 2014b), the Canadian Highway Bridge Design Code S6 (CSA 2014a) and the AASHTO
 246 specifications (AASHTO 2012) are based on the Modified Compression Field Theory (Collins et
 247 al. 1996; Bentz and Collins 2004). These requirements define the nominal shear strength
 248 attributed to the concrete, V_c , and the shear resistance provided by the shear reinforcement, V_s .
 249 The equations from CSA standard A23.3 (CSA 2014b), expressed in SI units, are given below
 250 for the nominal shear resistance (i.e., material resistance factors $\phi_c = \phi_s = 1$).

$$251 \quad V_c = \beta \sqrt{f'_c} b_w d_v \quad \text{in MPa units} \quad (5)$$

$$252 \quad \beta = \left(\frac{0.4}{1 + 1500 \varepsilon_x} \right) \left(\frac{1300}{1000 + s_{ze}} \right) \quad \text{mm} \quad (6)$$

$$253 \quad V_s = A_v f_y \frac{d_v \cot \theta}{s_v} \quad (7)$$

254 Where A_v is the area of transverse reinforcement within a distance s_v and β is the ability of the
 255 diagonally cracked concrete to resist shear by tension stiffening and aggregate interlock (Eq. (6)
 256). It is a function of the longitudinal strain, ε_x , at mid-depth of the beam and the equivalent
 257 horizontal crack spacing, s_{ze} . For concrete members without transverse reinforcement,
 258 $s_{ze} = 35d_v / (16 + a_g)$ (mm units). For members containing at least the minimum amount of
 259 transverse reinforcement, the equivalent crack spacing parameter s_{ze} is taken as 300 mm (12 in).
 260 This minimum amount of transverse reinforcement is determined from Eq. (8), where $C = 0.060$
 261 for both the CSA S6 code (2014a) and the CSA standard A23.3 (2014b), while $C = 0.083$, in
 262 MPa units for AASHTO specifications (2012).

$$263 \quad A_v \geq C \frac{\sqrt{f'_c}}{f_y} s_v b_w \quad (8)$$

264 The angle of principal compression in the concrete, θ , with respect to the longitudinal member
265 axis can be used to determine the number of effective transverse reinforcing bars, n_v , resisting
266 shear.

$$267 \quad n_v = \frac{d_v \cot \theta}{s_v} \quad (9)$$

268 ***Unstrengthened beams (Beams U1, U2 and U3)***

269 The calculated shear strength V_{calc} and the experimental shear capacity V_{exp} are given in Table 1.
270 The concrete nominal shear stress at failure ($v_c = V_{exp} / (b_w d_v)$) shows the size effect
271 phenomenon for beams without shear reinforcement. With an effective shear depth, d_v , of 333
272 mm and 359 mm respectively, beams U1 and U2 experienced a shear stress at failure of 1.59
273 MPa and 1.33 MPa, respectively, whereas the shear stress at failure of the deepest beam
274 specimens U3 ($d_v = 629$ mm) failed at a lower shear stress of 0.89 MPa. A good correlation
275 between experimental results and calculated values is achieved with the calculated shear
276 capacity, V_{calc} , for the beams without shear reinforcement being close to V_{exp} (average
277 $V_{calc} / V_{exp} = 1.00$).

278 ***Beam with conventional stirrups (Beam S1)***

279 For both tests on the beams S1-1 (tests S1 for the first loading and S1-1R for the reloading) with
280 stirrups, the predictions are very close to the experimental shear capacities. The average ratio
281 V_{calc} / V_{exp} is 1.05 while the predicted amount of shear reinforcing bars crossed by the diagonal
282 crack n_v is 2.35. These results are in good agreement with the cracking patterns shown in Fig. 6,
283 where the main diagonal crack crossed 3 and 2 bar locations for beams S1-1 and S1-1R,
284 respectively.

285 ***Beams with bonded shear reinforcing bars (Beams B1, B2, B3 and B4)***

286 For the beam specimens with epoxy bonded shear reinforcing bars, the predicted shear strengths
287 provided by the bonded shear reinforcing bars are determined from Eq. (7). The experimental
288 cracking patterns showed that the main diagonal crack crossed 2 reinforcing bar locations for
289 beams B2, B3 and B4-1 and 3 locations for beams B1 and B4-1R. Comparing these observations
290 with the values of n_v indicates reasonable predictions of the amount of transverse reinforcement
291 resisting shear. However, with the assumption that the added bars yield, the shear capacities of
292 beams B1, B2 and B3 are overestimated ($V_{calc} / V_{exp} = 1.28, 1.42$ and 1.41 , respectively) while for
293 beam B4 $V_{calc} / V_{exp} = 1.07$.

294 Previous measurements showed that bonded bars can fail by debonding before reaching f_y when
295 $\ell_e < \ell_d$ and hence some adjustments are necessary to account for this important effect. As shown
296 in Table 3, all the bonded shear reinforcing bars of beams B4 were able to fully develop their
297 yield strength and therefore, beam B4-1 and B4-1R had a similar behavior to the beam with
298 stirrups and the predicted shear capacity is close to the experimental shear capacity. For beams
299 B1, B2 and B3, a number of bonded bars have partially contributed to the shear capacity due to
300 the fact that a crack crossing a bar within ℓ_d may lead to the debonding of the bar. As illustrated
301 in Fig. 9, the length ℓ_y along a bar where a crossing diagonal crack would allow the
302 development of f_y in the bar can be defined as:

303
$$\ell_y = \ell_{bar} - 2\ell_d \quad (10)$$

304 Where ℓ_{bar} is the bar length and ℓ_d is the bar development length defined by Eq. (2). A diagonal
305 crack is more likely to cross within ℓ_y for small spacings and for long bonded bars. The bar

306 efficiency ratio in shear η can be defined as follows:

$$307 \quad \eta = \frac{\ell_y}{d_v} \quad (11)$$

308 By comparing beams with similar bar development lengths, the bars used for beam B1 and B2
309 ($\ell_{bar} = 345$ mm) are shorter than the bars used for beams B3 ($\ell_{bar} = 645$ mm) and B4 ($\ell_{bar} = 660$
310 mm). According to the ℓ_d values presented in Table 3, the bar efficiency ratio η of these beams
311 are 0.45, 0.64, 0.72 and 0.77, respectively. The longer bars used for beams B3 and B4 are
312 therefore more efficient than the bars used for beams B1 and B2.

313 A small shear reinforcement spacing ratio $k_v = s_v / d_v$ also enables the diagonal crack to cross a
314 larger number of shear reinforcing bars within the yielding length, ℓ_y . For beams B1, B2 and
315 B3, the s_v / d_v values were close to the maximum allowed by standards (0.75 for the S6 code
316 (CSA 2014a) and 0.80 for AASHTO (2012)). Consequently, the main shear cracks intercepted
317 two bar locations near their extremities. However, for beam B4, the smaller ratio k_v of 0.61
318 allowed the main diagonal crack to cross two bar locations within ℓ_y , with one being intercepted
319 at its mid-height.

320 ***Beam with overlapped bonded shear reinforcing bars (Beam B5)***

321 For the shear capacity of beam B5, the overlapped area of shear reinforcing bars is neglected
322 ($A_v = 400$ mm²). The experimental cracking patterns showed that the main diagonal crack
323 crossed 2 reinforcing bar locations, which is similar to the prediction of $n_v = 2.35$. However, the
324 prediction underestimates the shear capacity for beam B5-1 ($V_{calc} / V_{exp} = 0.85$). This
325 underestimation can be explained by the overlapping of the bonded bars. The lap length of 300
326 mm is longer than twice the development length ($2\ell_d = 182$ mm in the epoxy-filled hole). The bar

327 efficiency ratio η is 1.03 for each pair of overlapped bars and hence the effective bar capacity is
328 higher than $A_v f_y$. As shown in Table 3 for beam B5-1, the yield force of bars R3b was fully
329 developed while the bars R3t reached a maximum stress of 192 MPa ($0.426A_v f_y$) (Table 3).
330 Thus, the two bars R3t and R3b are able to carry a total of 256 kN ($1.426A_v f_y$) instead of 179 kN
331 ($A_v f_y$) assuming $A_v = 400 \text{ mm}^2$. In addition, beam B5 has shown a larger number of diagonal
332 cracks in the overlapped bar region than the other beams. The reduction of the crack spacing in
333 the lapped region would likely increase V_c for beam B5.

334 **Maximum spacing of added bonded bars**

335 The experimental results of the beams with added bonded shear reinforcing bars have shown that
336 the maximum bar spacing needs to be smaller than that required in current codes for stirrups.
337 According to current codes (CSA S6 (2014a), CSA A23.3 (2014b) and AASHTO (2012)),
338 stirrups shall be spaced so that every line inclined at an angle θ to the axis of the member and
339 extending toward the reaction from mid-depth to the member longitudinal flexural tension
340 reinforcement shall be crossed by at least one line of effective shear reinforcement. Over the full
341 shear depth, d_v , of the beam, a minimum of two stirrups intercepting the inclined compression
342 field are therefore required. This enables the development of the compression field between two
343 transverse bars carrying tension. The maximum spacing ratio $k_{v,max}$ and the maximum spacing
344 $s_{v,max}$ of transverse reinforcement can therefore be determined as follow:

$$345 \quad k_{v,max} = \frac{s_{v,max}}{d_v} = \frac{1}{2 \tan \theta} \quad (12)$$

346 For example, for an angle θ of 34° , $k_{v,max} = 0.75$ as defined by the Canadian Highway Bridge
347 Design Code S6 (CSA 2014a). For bonded bars, the location of the crack determines the bar

348 embedded length and the bar capacity. For a diagonal crack crossing within the bar yielding
 349 length ℓ_y , the transverse reinforcement can reach its yield strength. For cracks crossing within
 350 length ℓ_d , partial contribution of the bar is expected. By assuming a constant bond strength
 351 along the development length of the bars, the tensile stress f_{sv} that can be developed in the
 352 transverse bonded reinforcement can be determined by Eq. (13), where y is the smallest
 353 distance measured from the bar extremity (see Fig. 9).

$$354 \quad f_{sv} = \frac{4\tau_b y}{d_b} \leq f_y \quad (13)$$

355 The resulting tensile stress distribution in the bonded bars is illustrated in Fig. 9. To meet the
 356 maximum spacing requirement, this figure shows how the bonded bars can develop $A_v f_y$ in both
 357 lower and upper parts of the beam. The maximum spacing of transverse bonded reinforcement
 358 can therefore be determined by Eq. (14).

$$359 \quad s_{v,max} = k_{v,max} \ell_y \leq k_{v,max} d_v \quad (14)$$

360 Note that using $k_{v,max} \ell_y < s_v < k_{v,max} \ell_{bar}$ results in partial development of the vertical bars. By
 361 taking into account the efficiency ratio η from Eq. (11), the maximum spacing ratio of bonded
 362 transverse reinforcement can be rewritten as:

$$363 \quad \frac{s_{v,max}}{d_v} = \eta k_{v,max} \quad (15)$$

364 Where η is defined by Eq.(11) and is not greater than 1. Because ℓ_d is a constant for a given
 365 bonded bar, its efficiency ratio increases as the depth of the slab increases. For shallower slabs, it
 366 would be required to use mechanical anchorages at the extremities of the added bars to avoid
 367 debonding, or to use inclined bonded bars to increase their embedded length.

368 From Eq. (15) and $k_v = s_v / \ell_y$, $k_v = 1.60, 1.18, 1.04, 0.79$ and 0.61 for beams B1, B2, B3, B4 and
369 B5 respectively. Beams B4 and B5 are the only ones meeting the standards requirement ($k_{v,max} =$
370 0.75 and 0.80 for the CSA S6 code (2014a) and AASHTO specifications (2012), respectively)
371 while, the transverse bar spacing of the other beams enables partial development of the bonded
372 bars ($k_{v,max} \ell_y < s_v < k_{v,max} \ell_{bar}$). Consequently, the responses of beams B4 and B5 were similar to
373 the beam with stirrups while the shear capacities of the other beams were lower than the ones
374 predicted based on specifications for members with conventional stirrups.

375 **Conclusions**

376 Experiments were carried out on thick concrete slab strips (beams) to evaluate the efficiency of
377 different shear strengthening techniques. The shear capacities were compared to the predictions
378 using current design provisions for elements with or without conventional transverse
379 reinforcements (stirrups). The experimental results showed that:

- 380 1. The strengthening techniques resulted in increased shear and deflection capacities
381 compared to beams without shear reinforcement.
- 382 2. The beams with added epoxy-bonded bars experienced a rapid propagation of the critical
383 diagonal crack. This resulted in a decrease of stiffness compared with the beams
384 reinforced with conventional stirrups.
- 385 3. Short embedded bonded bars ($\ell_e < \ell_d$) failed by debonding at the ends of the bars before
386 reaching f_y and hence adjustments are needed to accurately predict the strength by using
387 current code provisions that assume yielding of the transverse reinforcement.
- 388 4. Closely spaced longer bonded bars are more likely to result in yielding of the added
389 epoxy-bonded bars. In such cases, beams exhibit a similar behavior to beams with

390 stirrups and the predictions using current code provisions accurately predict the shear
391 strength.

392 5. For added epoxy bonded bars, the maximum spacing required by the current codes for
393 conventional stirrups can result in an overestimation of the shear capacity of up to 48%.
394 Therefore, a maximum spacing requirement for transverse bonded bars has been
395 proposed.

396 6. The predicted shear capacities using current code provisions for the beams respecting the
397 proposed maximum spacing requirement of the transverse bonded reinforcement (Eq.
398 (15)) agree well with the experimental results.

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404 experiments were carried out in the structures laboratory at Université Laval.

405

406 **Notation**

407 The following symbols are used in this paper:

408 A_s = area of longitudinal reinforcement on the flexural tension side;

409 A_v = area of shear reinforcement within a distance s_v ;

410 b_w = beam web width;

411 d = effective depth to the main tension reinforcement;

412 d_b = reinforcing bar diameter;

413 d_v = effective shear depth, taken as the greater of $0.9d$ and $0.72h$;

414 E_s = modulus of elasticity of steel;

415 f_c' = concrete cylinder compressive strength;

416 f_{sv} = stress in transverse reinforcement;

417 f_y = yield strength of reinforcement;

418 k_v = spacing ratio of transverse reinforcement;

419 ℓ_{bar} = length of added shear reinforcing bar;

420 ℓ_d = bar tension development length;

421 ℓ_e = bar embedded length;

422 ℓ_y = bar yielding length, equal to $\ell_{bar} - 2\ell_d$;

423 n_v = number of effective transverse reinforcing bars;

424 s_v = spacing of transverse reinforcement taken along the member longitudinal axis;

425 s_{ze} = equivalent longitudinal crack spacing;

- 426 V_c = shear resistance attributed to the concrete;
- 427 V_s = shear resistance provided by shear reinforcement;
- 428 β = factor accounting for shear resistance of cracked concrete;
- 429 γ_c = concrete density (kN/m³);
- 430 ε_{sv} = strain in transverse reinforcement;
- 431 ε_x = longitudinal strain at mid-depth of the member;
- 432 η = bonded bar efficiency ratio in shear;
- 433 θ = angle of diagonal compressive stresses to the longitudinal axis of the member;
- 434 ρ = longitudinal reinforcement ratio, equal to $A_s / b_w d$ for a rectangular beam; and
- 435 τ_b = bond strength of the adhesive (MPa).

436

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Table 1. Main properties of tested specimens and comparison to the predicted shear capacities.

Beam designation	Strengthening technique	h mm	d mm	ρ %	d_v mm	A_v mm ²	s_v mm	s_v/d_v	γ_c kN/m ³	f_c MPa	V_{exp} kN	δ_{exp} mm	V_c kN	n_v	V_s	V_{calc} kN	V_{calc}/V_{exp}	
Strengthened slabs																		
B1	1	Bonded	450	370	3.10	333	400	240	0.72	22.3	31.7	13.7	212	2.03	390	602	1.28	
																		2
B2	1	Bonded	450	399	2.05	359	200	260	0.72	22.5	34.3	8.4	248	2.05	179	426	1.48	
																		2
B3	1	Bonded	750	699	1.17	629	400	470	0.75	22.7	34.0	12.3	349	1.82	350	699	1.42	
																		2
B4	1	Bonded	750	694	1.65	625	400	380	0.61	22.6	34.5	12.0	389	2.34	420	809	1.09	
																		1R
B5	1	Overlapped	750	694	1.65	625	400	380	0.61	22.6	32.6	15.6	380	2.35	421	801	0.85	
																		1R
S1	1	Stirrups	750	694	1.65	625	400	380	0.61	22.2	33.3	10.7	383	2.35	421	804	1.11	
																		1R
Average																1.22		
CoV																0.17		
Unstrengthened slabs																		
U1	1	none	450	370	3.10	333	-	-	-	22.3	31.1	7.6	285	-	-	285	0.88	
																		2
U2	1	none	450	399	2.05	359	-	-	-	22.5	34.9	6.9	287	-	-	287	1.03	
																		2
U3	1	none	750	699	1.17	629	-	-	-	22.4	35.8	5.0	389	-	-	389	1.13	
																		2
Average																1.00		
CoV																0.11		

Note: beams U1, U2, B1 and B2 are loaded at one-third span, while other beams are loaded at mid-span. To distinguish between loading and reloading tests of the same beam, reloaded beams are identified "R". For all beams, width $b_w = 610$ mm

*This beam did not experience shear failure. The capacity was limited by the steel clamped assemblies used to strengthen beam S1-1

Table 2. Steel reinforcing bars properties

Test Series	Bar Designation	d_b (mm)	A_b (mm ²)	f_y (MPa)	f_u (MPa)
1st	10M	11.3	100	436	632
	15M	16.0	200	480	690
	25M	25.2	500	468	660
2nd	15M	16.0	200	448	633
	30M	29.9	700	508	668

Table 3. Bonded bars embedded length and bar stress

Beam	Bars	ℓ_e (mm)	ℓ_d (mm)	L_{tc} (mm)	$f_{sv,calc}$ ^b (MPa)	$f_{sv,exp}$ ^c (MPa)	
B1	1	R2	50	97.7	4	248	332
		R3	114		42	480	480
		R4	84		30	410	180
	1R	R2	10	97.2	62	51	N/A
		R3	153		20	480	N/A
		R4	85		16	420	N/A
B2	1	R3	93	62.4	20	436	436
		R4	26		0 ^a	178	272
	1R	R3	139	62.2	31	436	436
		R4	45		28	315	270
B3	1	R2	88	97.1	42 ^a	433	378
		R3	119		15	480	480
	2	R2	27	96.2	95	136	144
		R3	119		11	480	480
		R4	124		96	448	61
B4	1	R3	308	90.5	88	448	448
		R4	128		92 ^a	448	374
	1R	R2	135	90.5	146	448	431
		R3	327		107	448	448
		R4	124		96	448	61
B5	1	R3b	129	91.0	195 ^a	448	448
		R3t	39		146	192	N/A
		R4t	172		17	448	448
	1R	R2b	196	91.0	194	448	448
		R2t	35		110	171	N/A
		R3b	16		169	77	N/A
		R3t	202		21	448	448
		R4t	111		75	448	439

^a Several cracks cross the transverse reinforcing bar. The bar embedded length and the relative distance to the strain gage are not measured with the same crack.

^b Determined with Eq. 3.

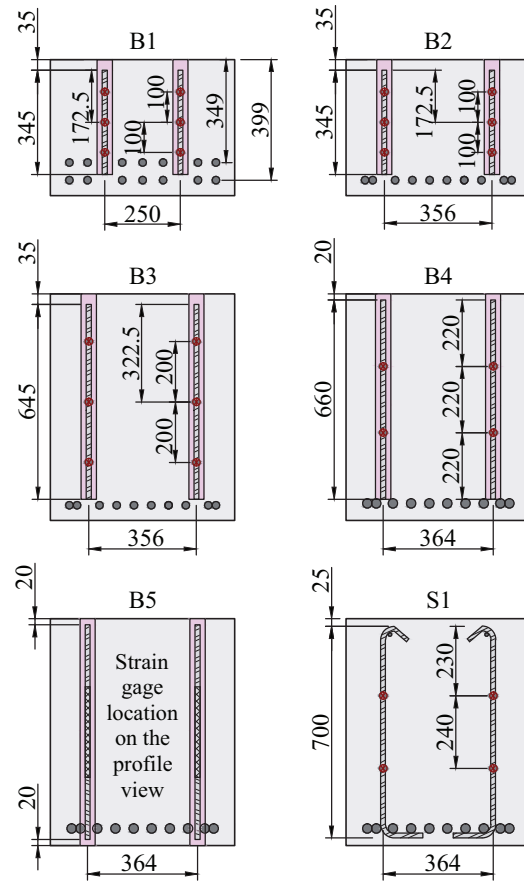
^c Determined with Eq. 4 and the maximum measured bar strain in Fig. 8.

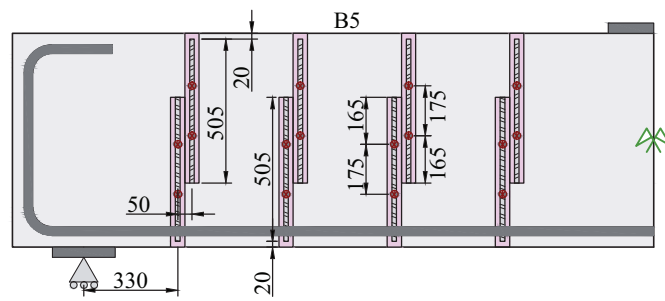


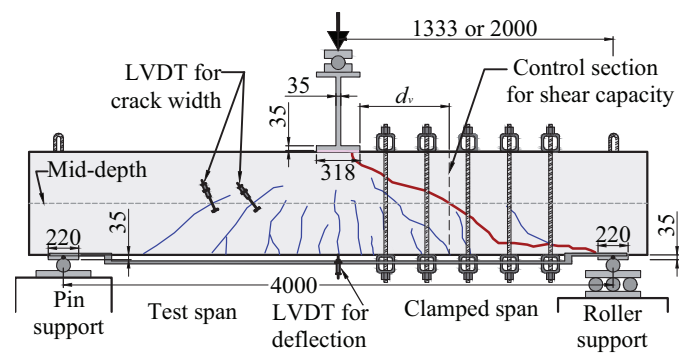


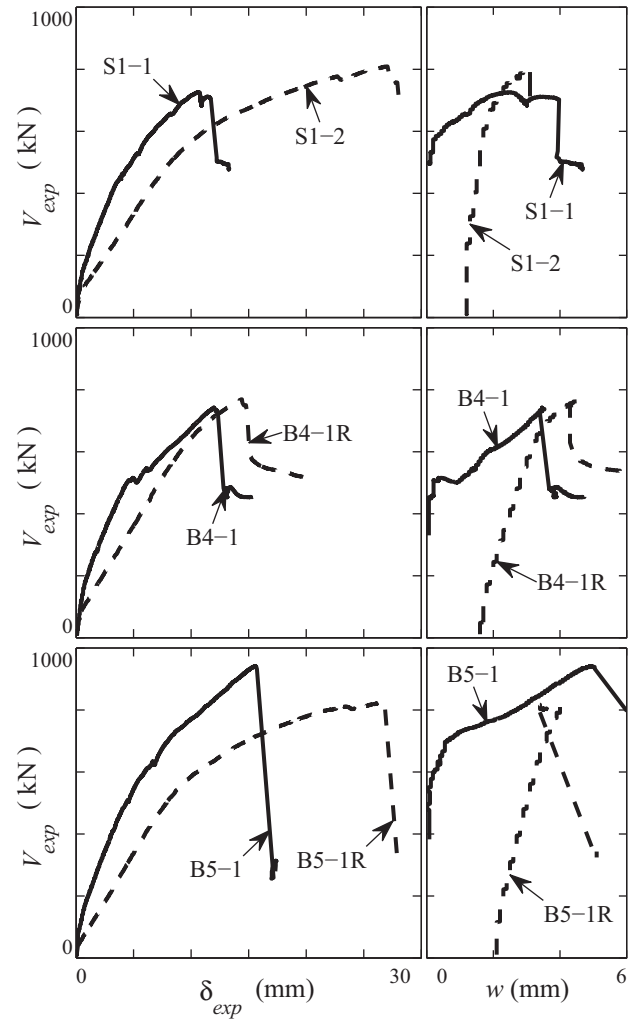
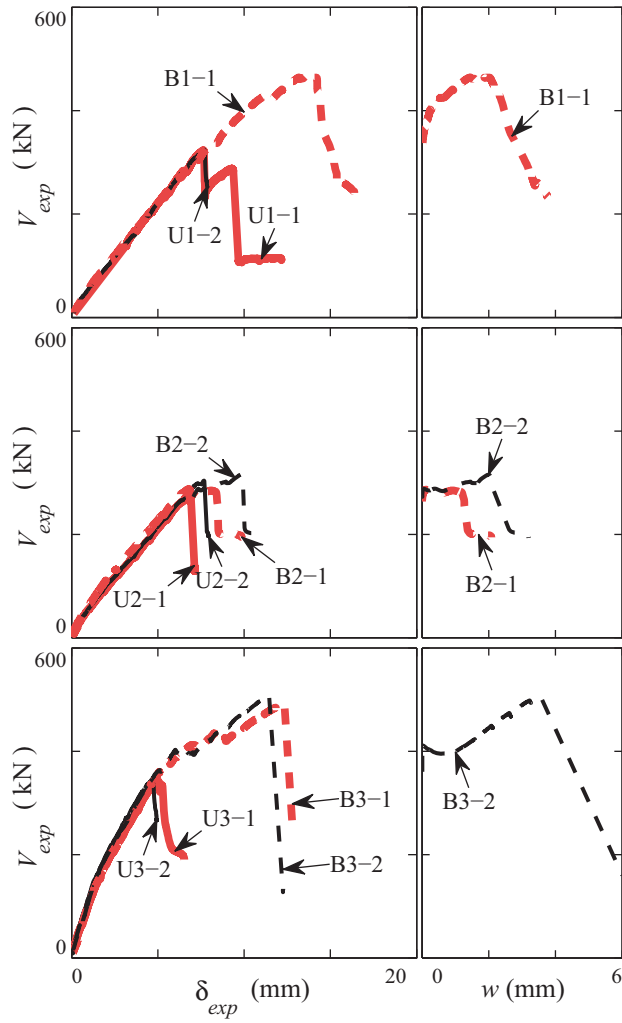


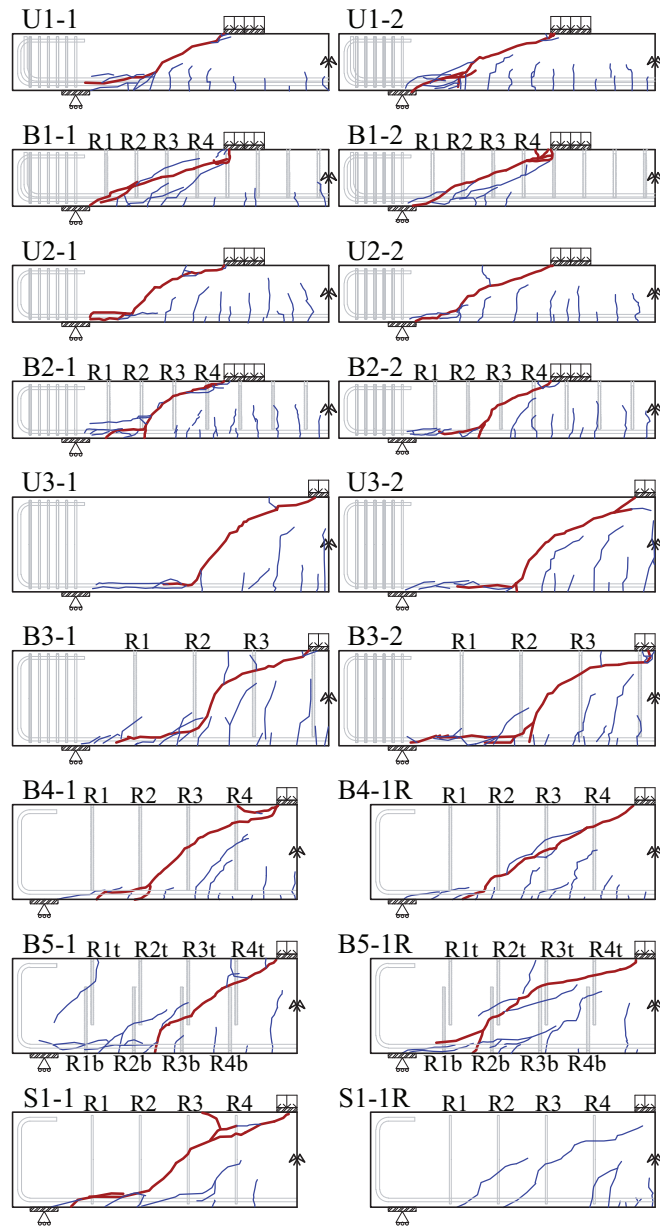


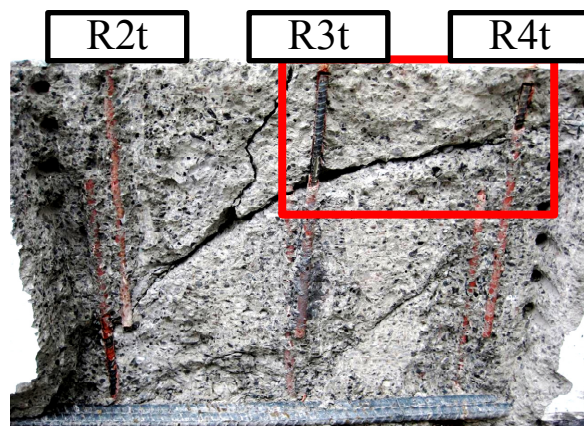


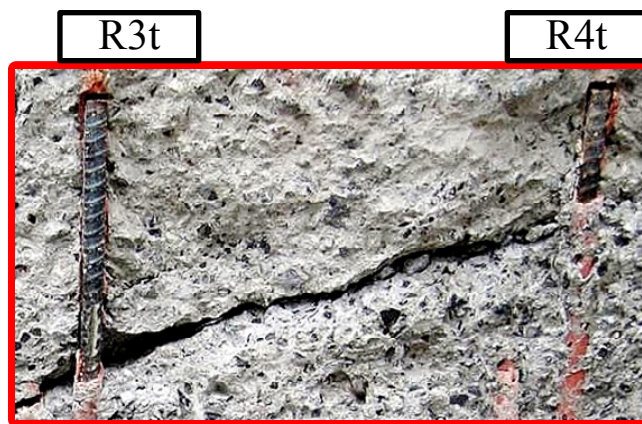


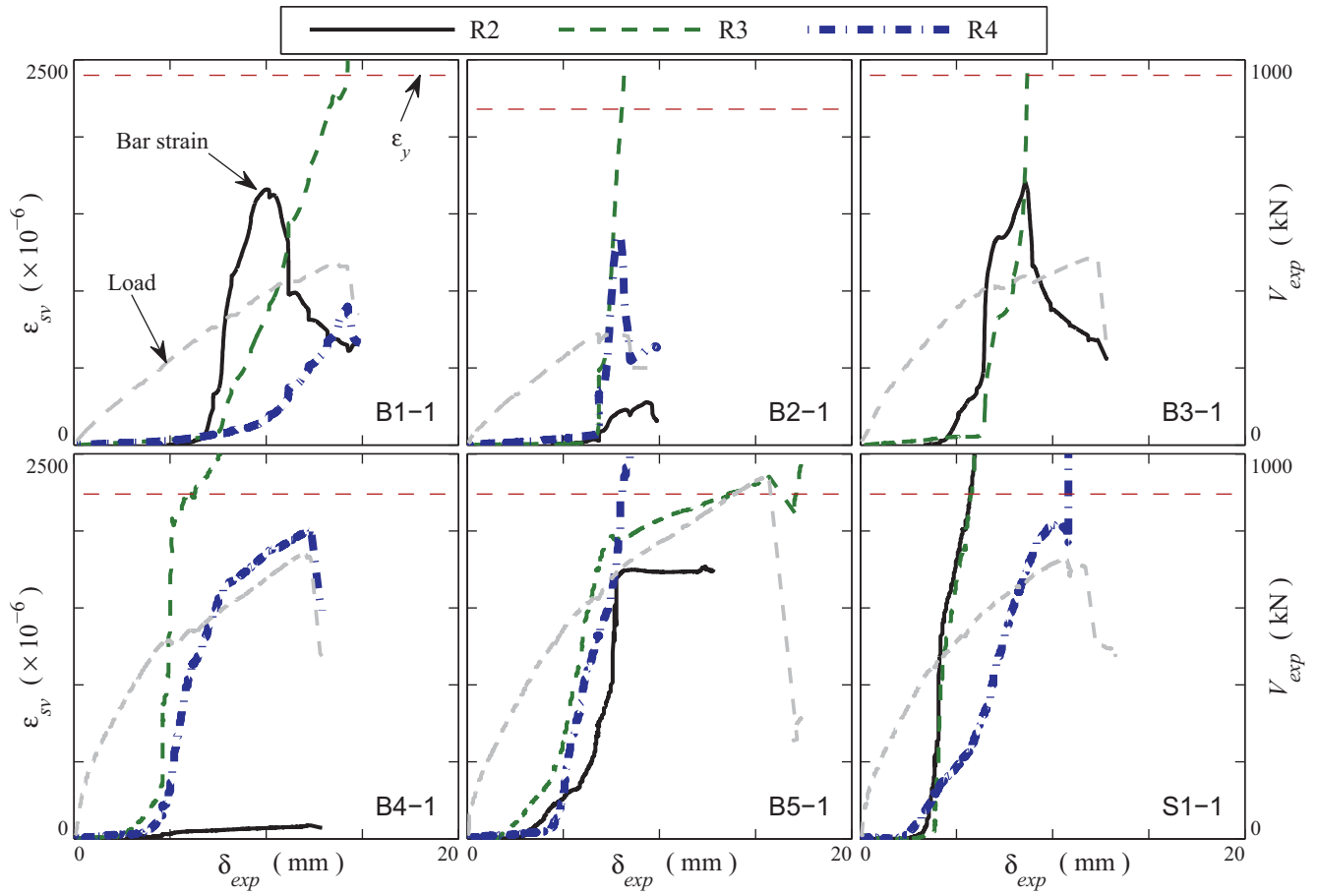


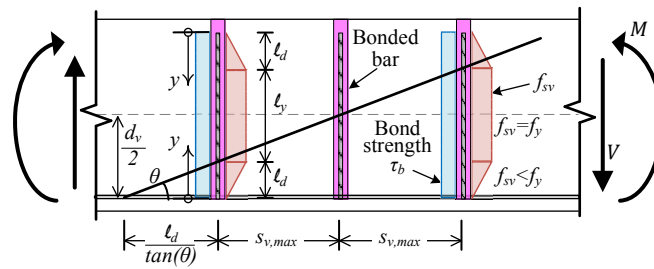












1 **Figure caption list**

2 **Fig. 1.** Partial collapse of south portion of Concorde Overpass due to shear failure of thick slab
3 (a) Aerial view of south portion (with permission Johnson et al. 2007) and (b) Shear failure of
4 thick slab that started near southeast corner (image by D. Mitchell)

5 **Fig. 2.** Installation of the epoxy bonded bars (a) from the top beam surface and (b) from the
6 bottom beam surface (image by J. Bastien)

7 **Fig. 3.** (a) Tested strengthened slab slices specimens (b) Profile view of beam B5 (dimensions in
8 mm)

9 **Fig. 4.** Experimental setup for reloading (R) stage (dimensions in mm)

10 **Fig. 5.** Load V_{exp} and critical shear crack width w vs beam deflection curves δ_{exp}

11 **Fig. 6.** Cracking pattern of tested beam

12 **Fig. 7.** Profile cut section of beam test B5-1R (a) Overall view of internal shear crack (b) Close-
13 up of shear failure crack (image by J. Bastien)

14 **Fig. 8.** Strain of shear reinforcing bars near the main shear crack and applied load vs beam
15 deflection (for beam B5-1, bars R2, R3 and R4 refer to R2b, R3b and R4t)

16 **Fig. 9.** Maximum bar spacing for beam with bonded shear reinforcing bars

17